Hunan University

37th International Conference on Boundary Elements and Other Mesh Reduction Methods

8 – 10 September 2014

An Adaptive Element Subdivision technique for Evaluating 3D Weakly Singular Boundary Integrals

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Outline

- Introduction
- Adaptive element subdivision technique
- Numerical examples
- Conclusions

Introduction

we consider the boundary integral equations of 3D potential problems in the domain Ω encolsed by the boundary Γ .

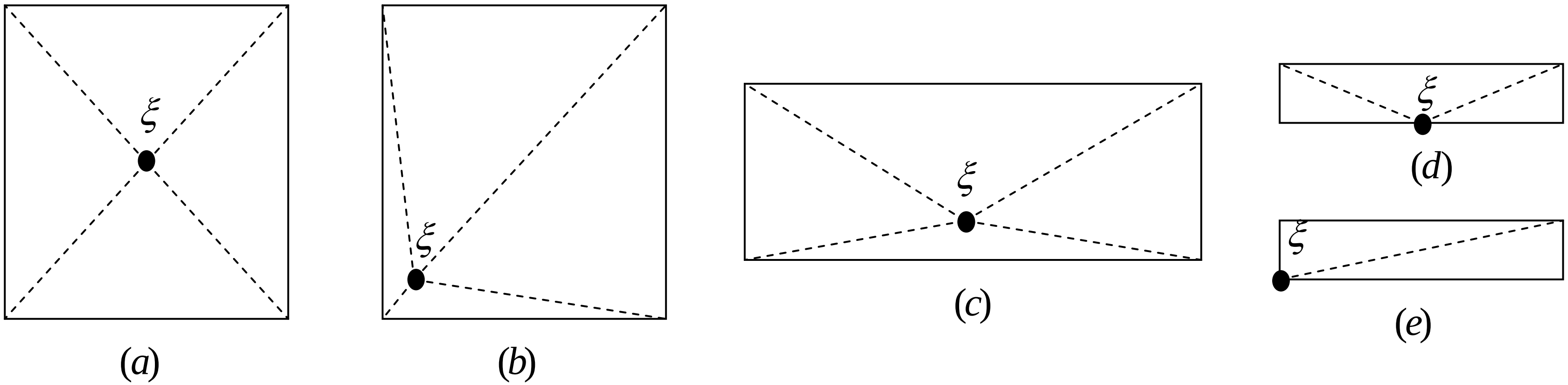
$$c(\mathrm{y})u(\mathrm{y}) = \int_{\Gamma} q(\mathrm{x})u^{*}(\mathrm{x},\mathrm{y})d\Gamma(\mathrm{x}) - \int_{\Gamma} u(\mathrm{x})q^{*}(\mathrm{x},\mathrm{y})d\Gamma(\mathrm{x})$$

$u^{*}(\mathrm{x},\mathrm{y})$is the fundamental solution for the 3D problem expressed as：

$$u^{*}(\mathrm{x},\mathrm{y}) = \frac{1}{4\pi}\frac{1}{r(\mathrm{x},\mathrm{y})}$$

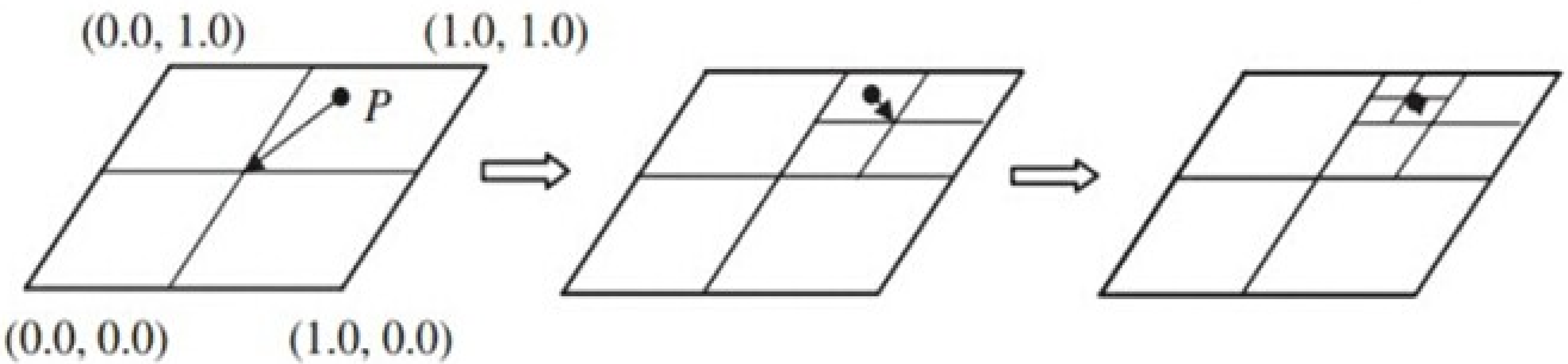
weakly singular integrals arise when the source point locates on the element.

Introduction



In the conventional method for solving weakly singular integrals, patches are obtained simply by connecting singular point with each vertex of the element.

Introduction

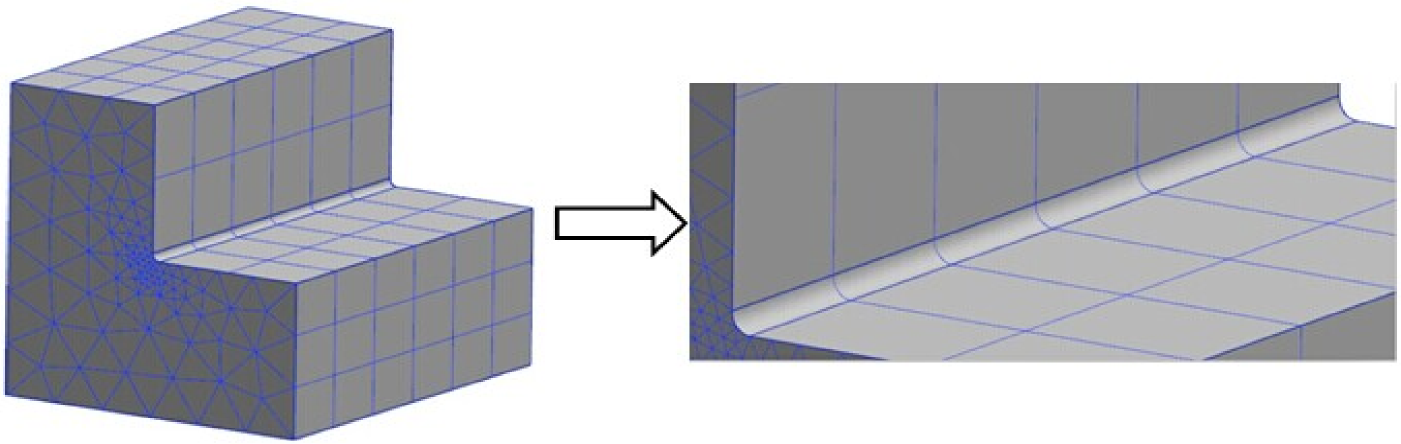


The Quadtree subdivisions of quadrilateral element in the element local coordinate space corresponding to an evaluation point *p*.

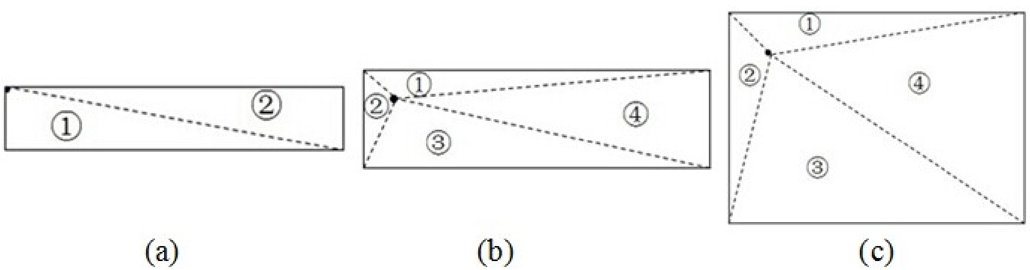
Introduction

- All the above mentioned methods are performed in the local coordinate system of the element rather than in the physical coordinate system.
- they may produce patches in “bad” shapes in case the element are curved or distorted or the element is irregular in shape.
- patches in good shape in the parametric space may become bad when they are mapped into the physical coordinate system.

Introduction



Slender curved element on the filleted corner.

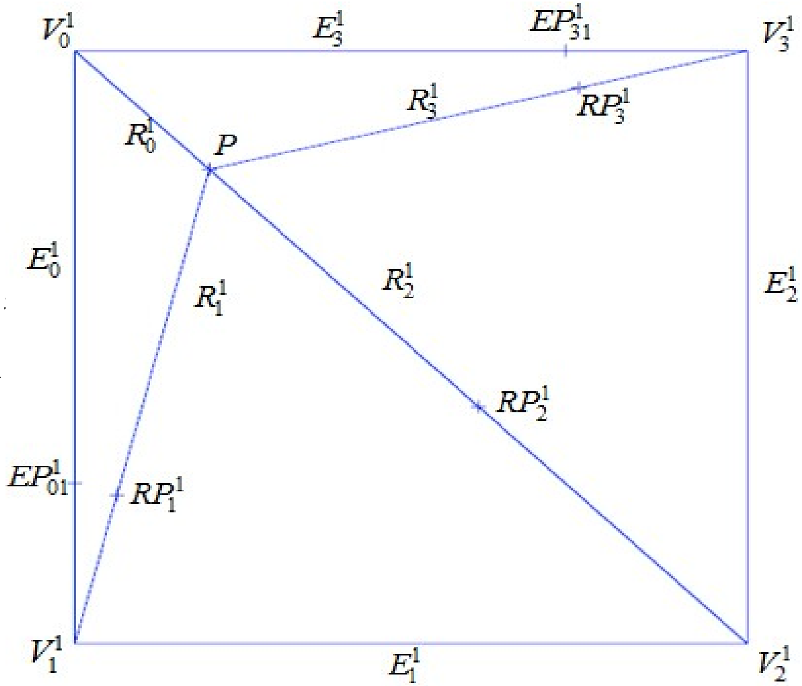


Patches obtained by the conventional method

Numerical tests have demonstrated that, with the same number of Gaussian sample points (6×6=36), the accuracy of integration with the 1/r kernel on patches (b)-①, (b)-②, (b)-③, (c)-① and (c)-② is bigger than 1%, which is completely wrong.

Adaptive element subdivision technique

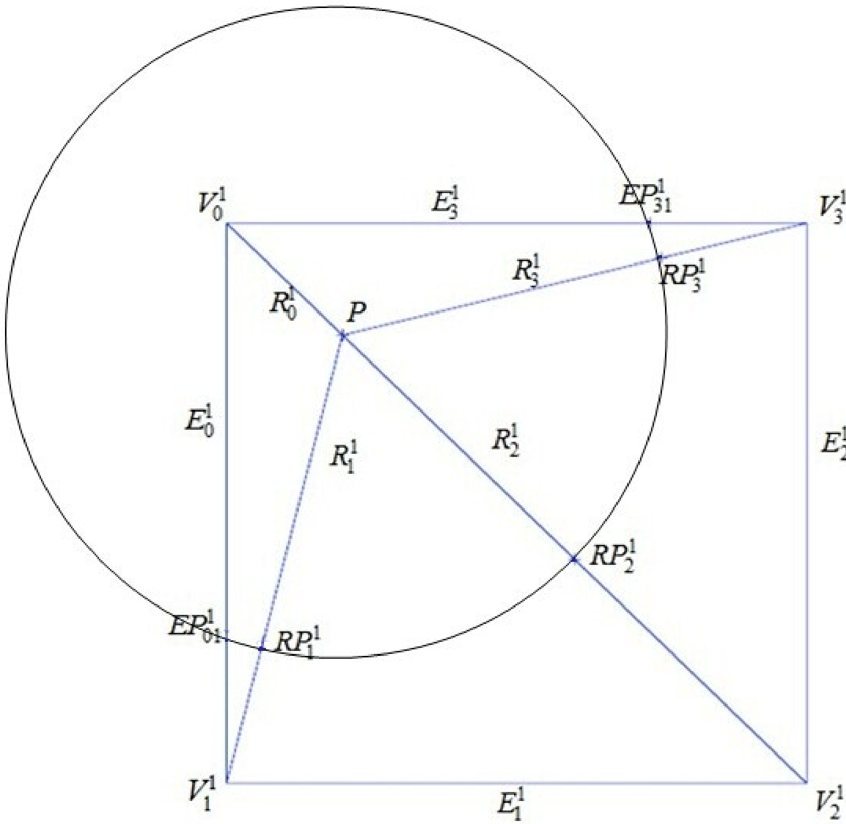
- P——the source point;
- V_i^j——the i-th vertex in the j-th step;
- E_i^j——the i-th edge in the j-th step;
- R_i^j——the line segment connecting P and V_i^j .
- EP_{ik}^j——the k-th intersection of with the j-th sphere, k=1, 2;
- RP_i^j——intersection ofwith the j-th sphere;
- L_i^j ——distance between P and V_i^j;
- D_i^j ——distance between P and V_i^j;
- Let $L_{\max} = \max\{L_i^j, D_i^j\}, L_{\min} = \min\{L_i^j, D_i^j\}(L_{\min} \neq 0)$

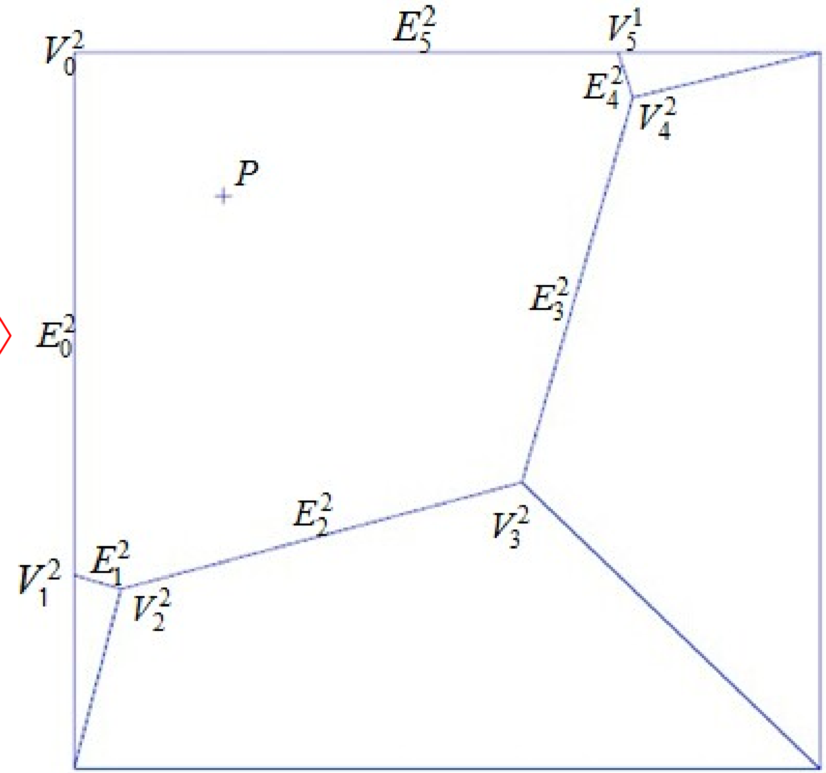


First step of element subdivision

$$r_j = \eta^j * L_{\max}$$

Adaptive element subdivision technique

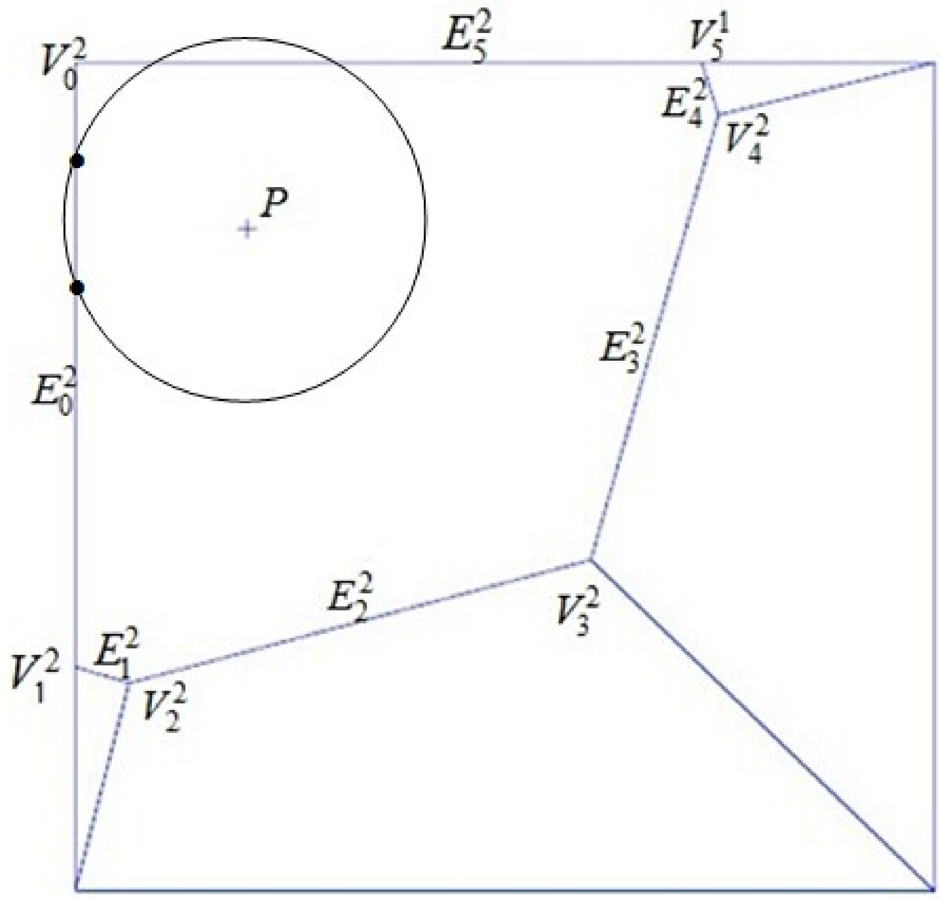




A sphere is constructed centered on P.

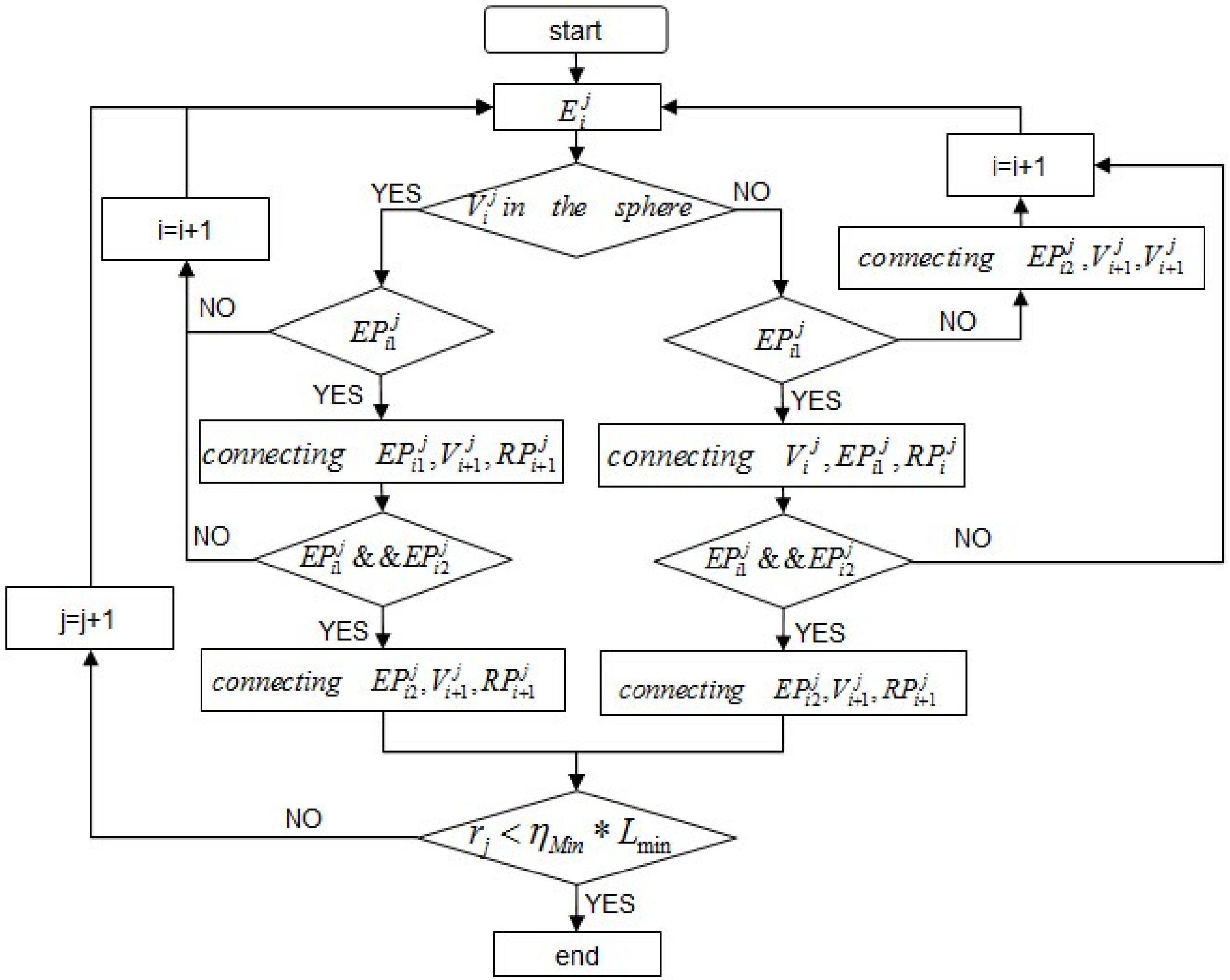
Patches obtained after the first step.

Adaptive element subdivision technique



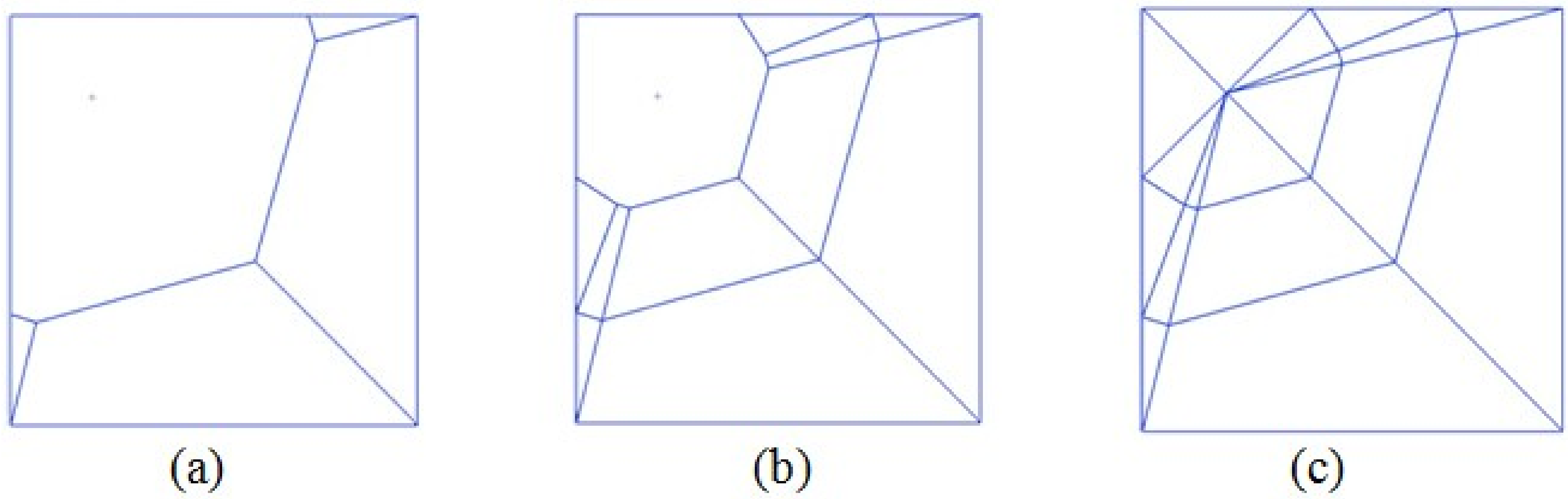
The situation we are trying to avoid that the two intersections are too close.

Adaptive element subdivision technique



Flow chart of subdivision algorithm.

Adaptive element subdivision technique



Patches obtained in subdivision steps: (a) the first step; (b) the second step; (c) the last step.

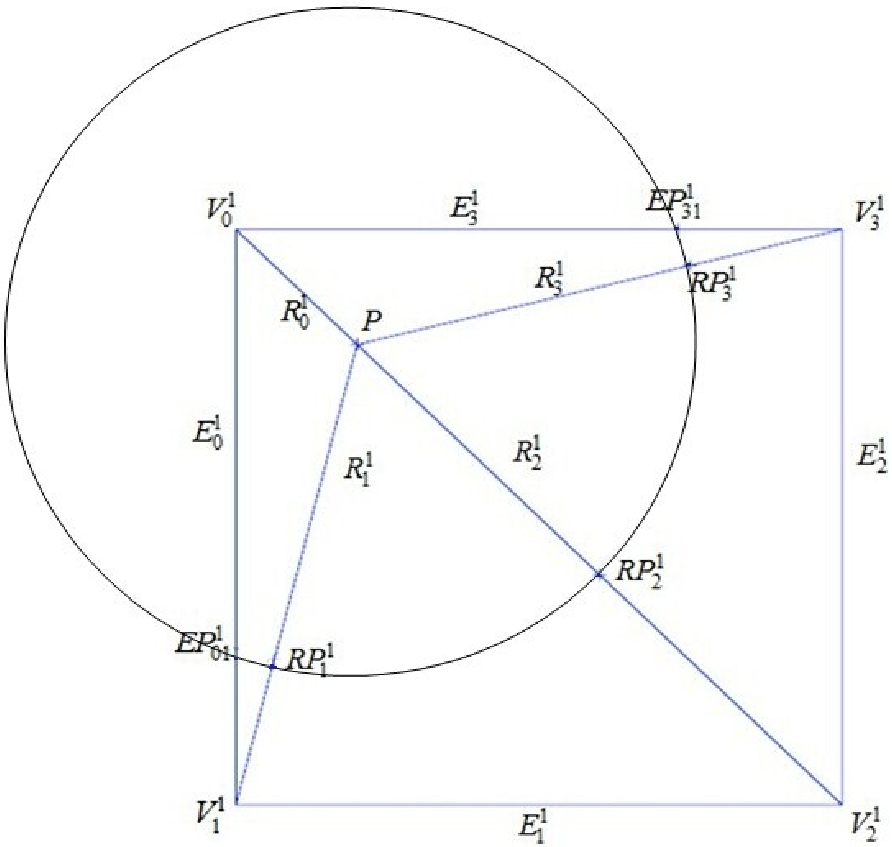
Drawback:

- **total number of patches is very large which will cause much greater computational cost.**
- **shapes of some patches without containing the source point are “bad”.**

merger operation

Adaptive element subdivision technique

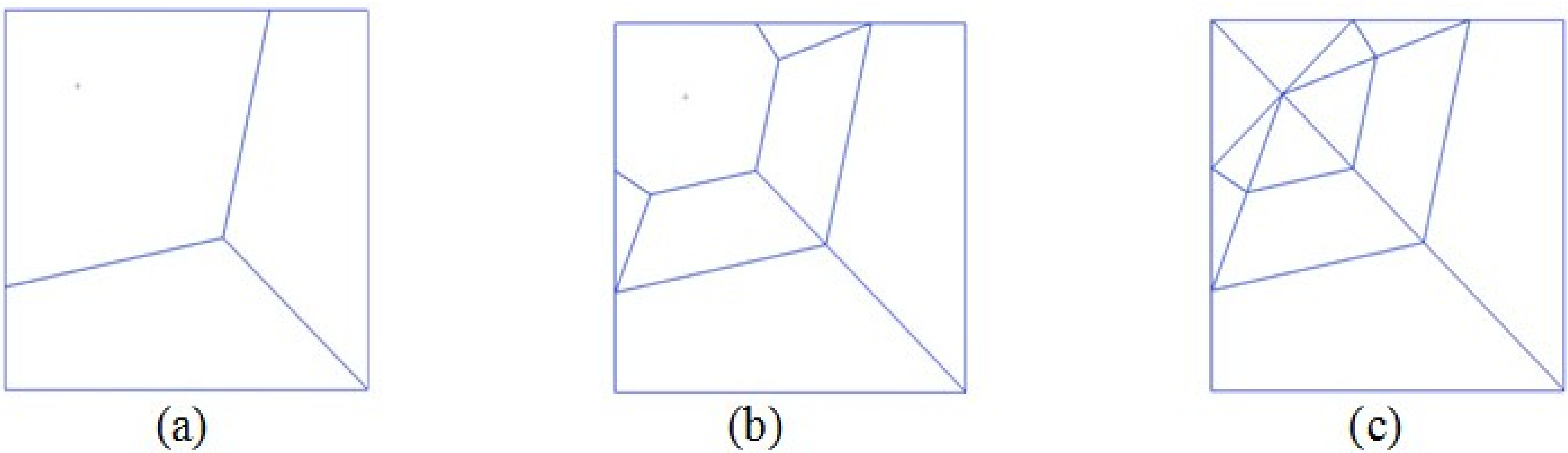
merging operation



- If the distance between EP_{ik}^j and the vertex V_i^j is smaller than an expected value, EP_{ik}^j will be moved to the adjacent vertex V_i^j.
- If the distance between RP_i^j and the vertex V_i^j is smaller than an expected value, RP_i^j will be moved to the adjacent vertex V_i^j.
- If the distance between EP_{ik}^j and RP_i^j is smaller than an expected value, EP_{ik}^j will be moved to RP_i^j.

Adaptive element subdivision technique

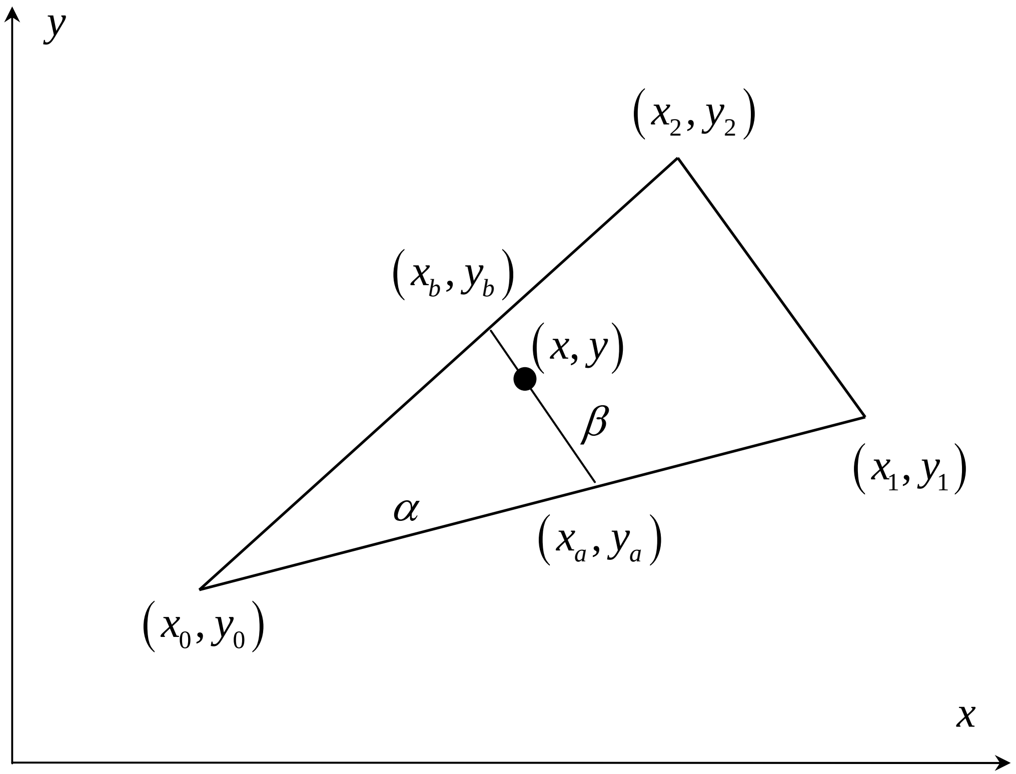
merging operation



Element subdivision after merging operation: (a) the first step; (b) the second step; (c) the last step.

After the merging operation, new patches are obtained, the number of patches is reduced and the shapes of patches are improved obviously compared with that in before.

Adaptive element subdivision technique



$$\begin{cases} x_a = x_0 + (x_1 - x_0)\alpha \\ y_a = y_0 + (y_1 - y_0)\alpha \end{cases}$$

$$\begin{cases} x_b = x_0 + (x_2 - x_0)\alpha \\ y_b = y_0 + (y_2 - y_0)\alpha \end{cases}$$

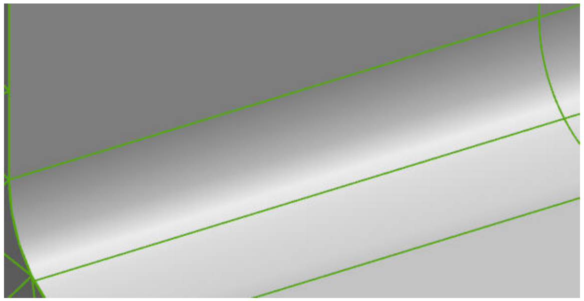
$$\begin{cases} x = x_a + (x_b - x_a)\beta \\ y = y_a + (y_b - y_a)\beta \end{cases} \quad \alpha, \beta \in [0,1]$$

$$\begin{cases} x = x_0 + (x_1 - x_0)\alpha + (x_2 - x_1)\alpha\beta \\ y = y_0 + (y_1 - y_0)\alpha + (y_2 - y_1)\alpha\beta \end{cases}$$

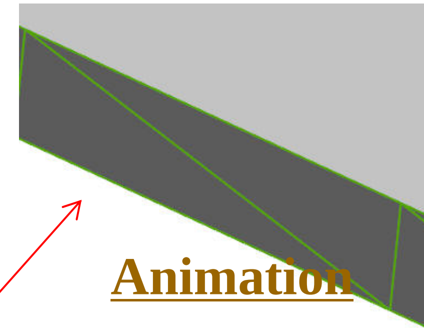
$$S_{\triangle} = \left| x_0 y_1 + x_1 y_2 + x_2 y_0 - x_0 y_2 - x_1 y_0 - x_2 y_1 \right|$$

it can be noted that the new coordinate system is much simpler to implement than the polar coordinate system. This is due to the fact that both α and β are constrained to the interval [0, 1] in each triangle, thus there is no need to calculate their spans.

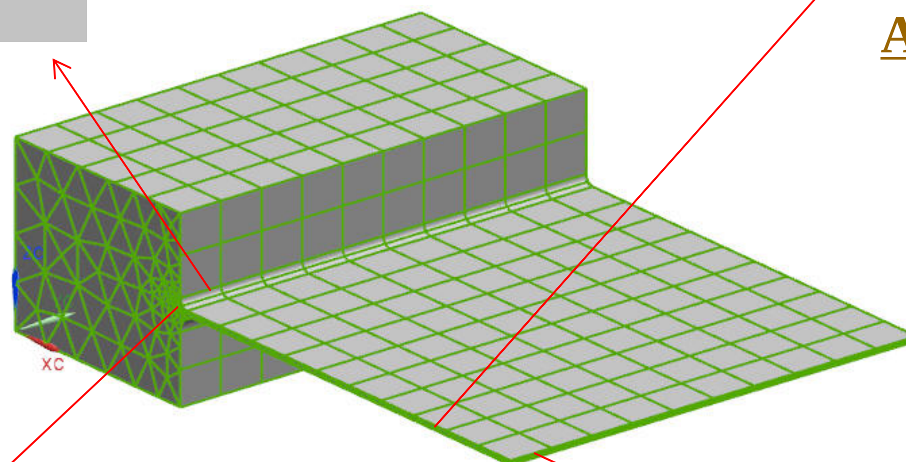
Adaptive element subdivision technique

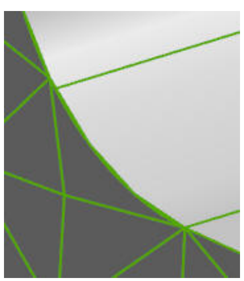


Animation

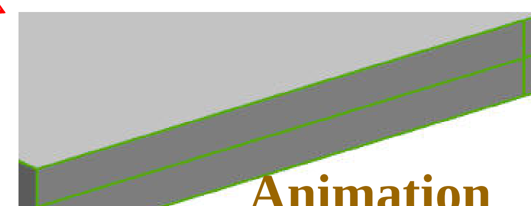


Animation





Animation



Animation

Numerical examples

the relative error is defined as follows:

$$\textbf{Relative Error} = \left| \frac{I_n - I_e}{I_e} \right|$$

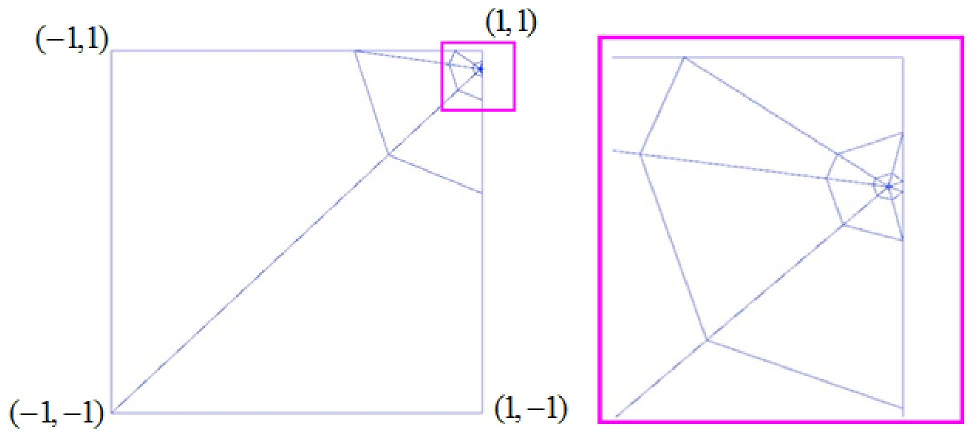
We consider the numerical evaluation of the integral

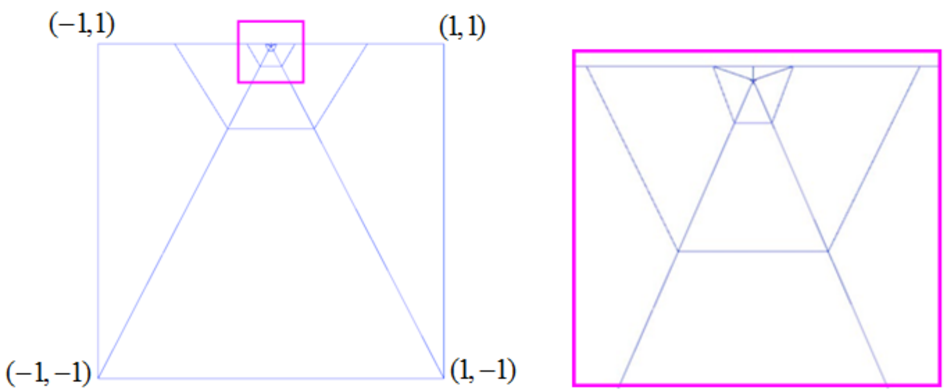
$$I = \int_{\Gamma} \frac{1}{4\pi r} d\Gamma$$

In all the numerical examples, the α - β transformation is used to remove singularities in the patches which contain the source point, while the remaining regular quadrilateral and triangular patches are respectively evaluated by the standard Gaussian quadrature and Hammer quadrature.

Numerical examples

Examples of planar element

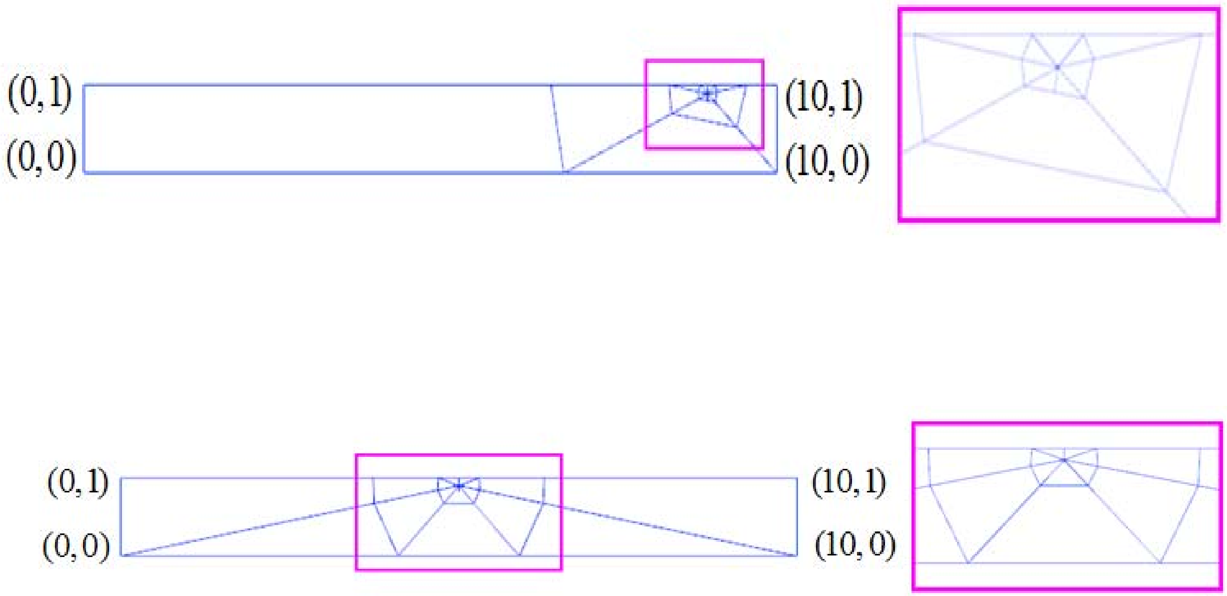




The sphere adaptive subdivisions of planar rectangular element.

Numerical examples

Examples of planar element



The sphere adaptive subdivisions of planar slender element.

Numerical examples

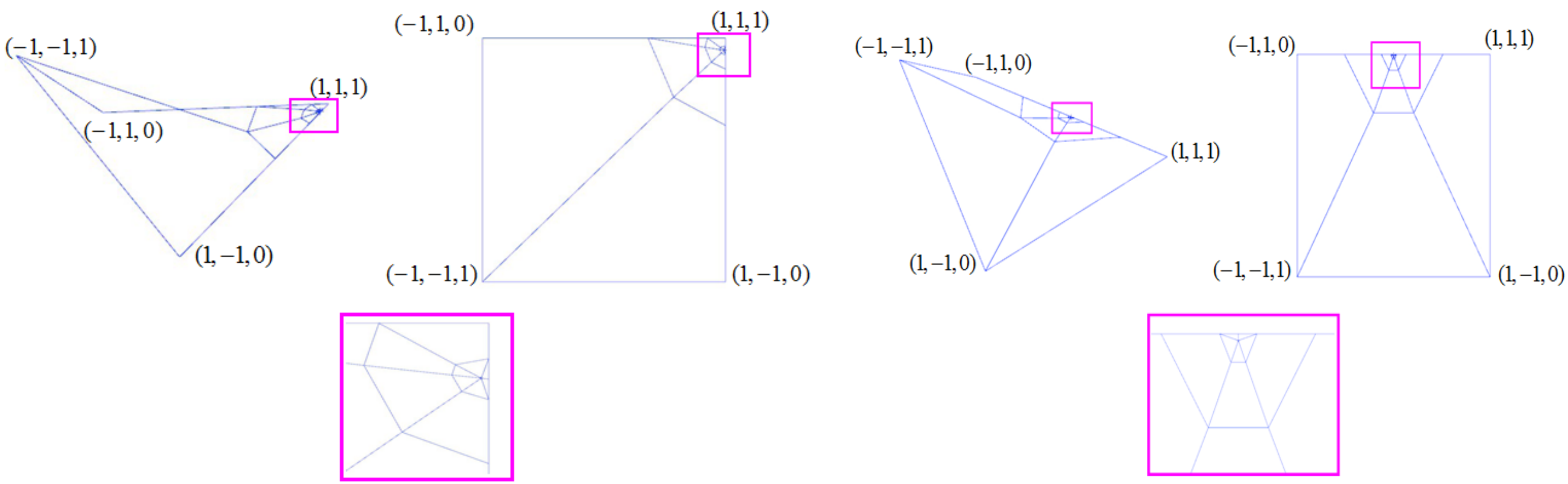
Examples of planar element

Table : Numerical evaluation for planar rectangular element and slender element.

Planar element	source point	Gaussian points number		Relative Error	
		direct subdivision	sphere subdivision	direct subdivision	sphere subdivision
rectangular	(0.99, 0.9)	1200	1191	4.46e-004	7.73e-008
	(0.0, 0.99)	1200	843	2.10e-003	3.35e-008
Slender	(9.0,0.9)	1200	1100	3.73e-004	2.73e-007
	(5.0, 0.9)	1200	1100	3.55e-003	8.18e-008

Numerical examples

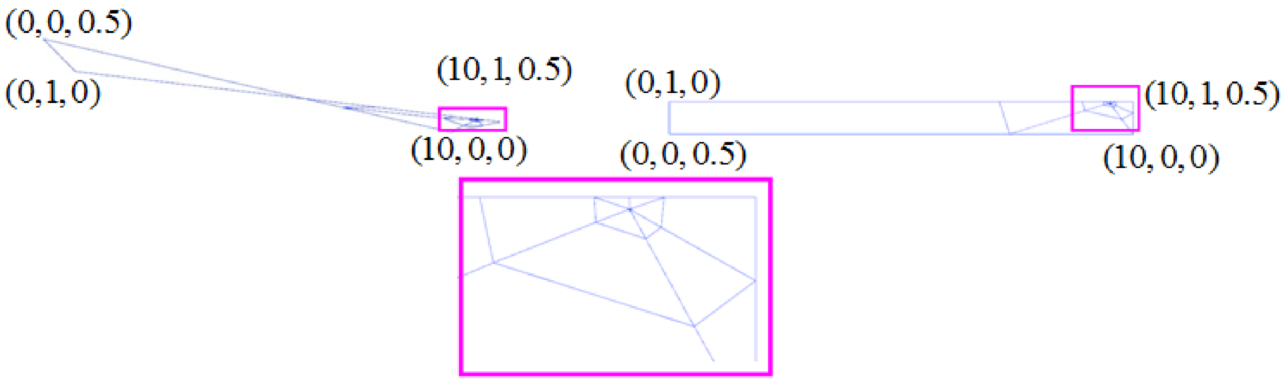
Examples of curved surface element

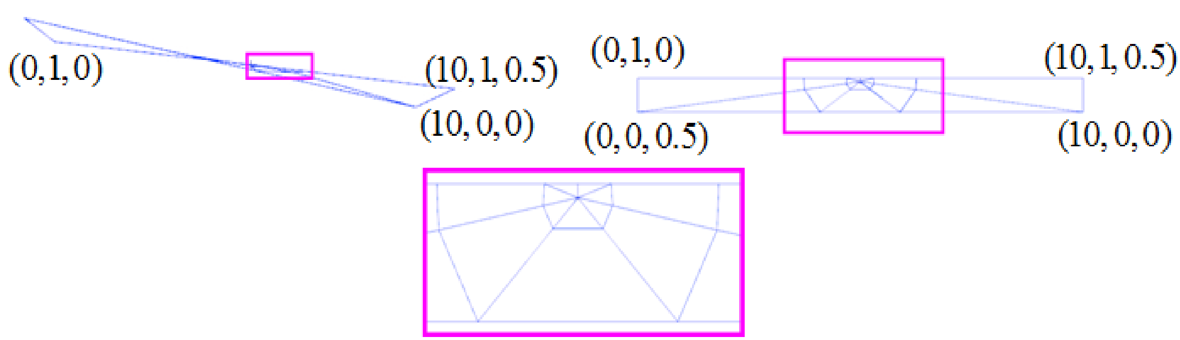


The sphere adaptive subdivisions of curved rectangular element.

Numerical examples

Examples of curved surface element





The sphere adaptive subdivisions of curved slender element.

Numerical examples

examples of curved surface element

Table 2: Numerical evaluation for curved surface rectangular and slender element

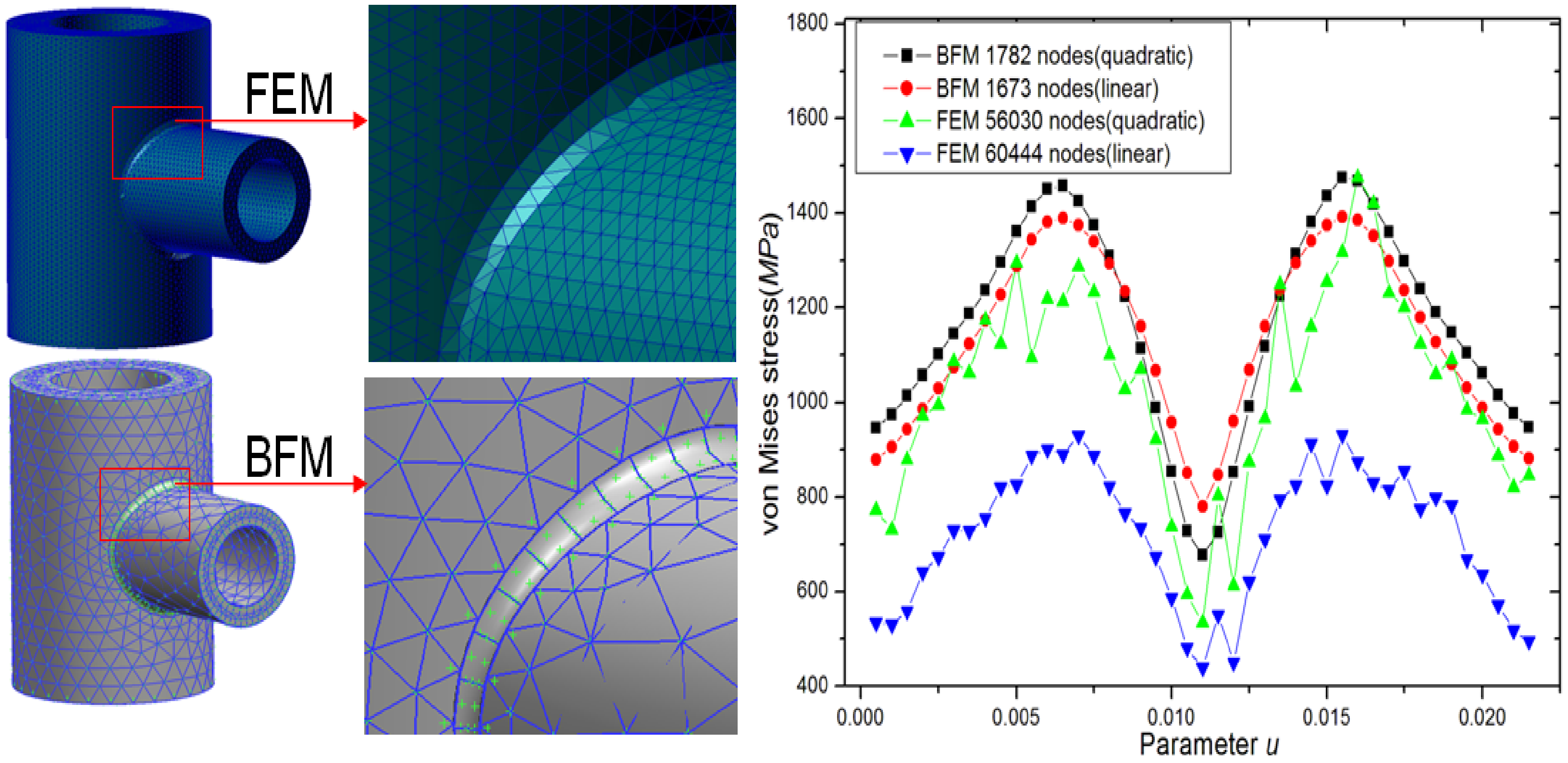
Curved surface element	source point	Gaussian points number		Relative Error	
		direct subdivision	sphere subdivision	direct subdivision	sphere subdivision
rectangular	(0.99,0.9,0.9455)	1200	815	5.74e-004	2.13e-007
	(0.0, 0.99,0.5)	1200	843	2.33e-003	8.44e-009
Slender	(9.5,0.95,0.4525)	1200	1045	1.28e-003	1.74e-007
	(5.0, 0.9,0.25)	1200	1100	3.50e-003	8.63e-008

The results demonstrate that shape of sub-elements for planar or curved surface element is good with the proposed adaptive element subdivision technique and our method can provide higher accuracy and efficiency than the conventional method with fewer Gaussian points.

Conclusions

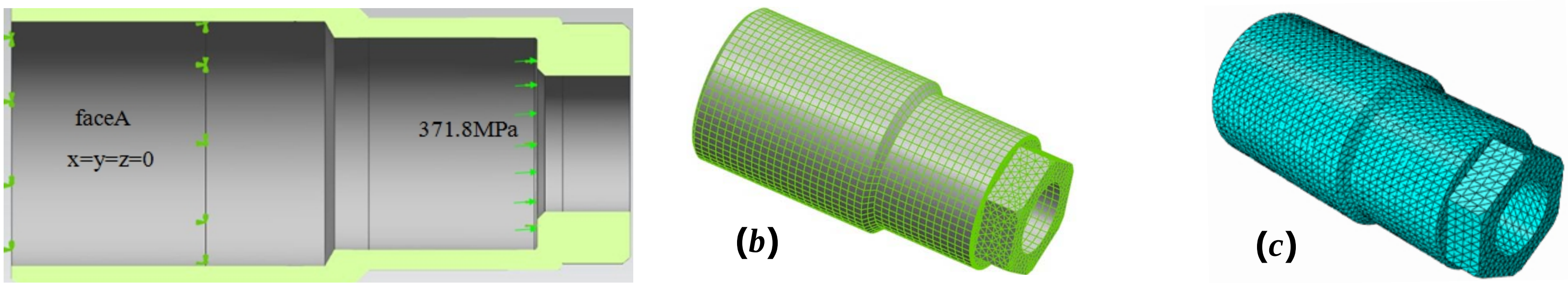
- A general adaptive element subdivision technique for the numerical evaluation of weakly singular integrals on 3D boundary element was proposed. Employing the proposed method, no matter planar and curved surface boundary element or where the position of singular point is located, sub-elements with good shape can be obtained which is convenient for getting higher accuracy. This algorithm is simple, powerful, initiative, versatile and useful where the best balance between accuracy and efficiency is achieved.
- An element is subdivided into a number of patches through a sequence of spheres with decreasing radius, and the obtained patches are automatically refined as they approaching the source point. Therefore, each patch is ensured to be “good” in shape and size for standard Gaussian quadrature.
- Our method is applicable to any shape of elements, no matter where the source point is located, namely, it’s suitable for both continuous and discontinuous elements.
- Numerical examples were presented to verify our method. Results demonstrated that with this adaptive element subdivision algorithm higher accuracy and efficiency can be obtained than the conventional method.
- Extension of our work to 3D nearly singular integral is ongoing now.

■ Manifold with fillet



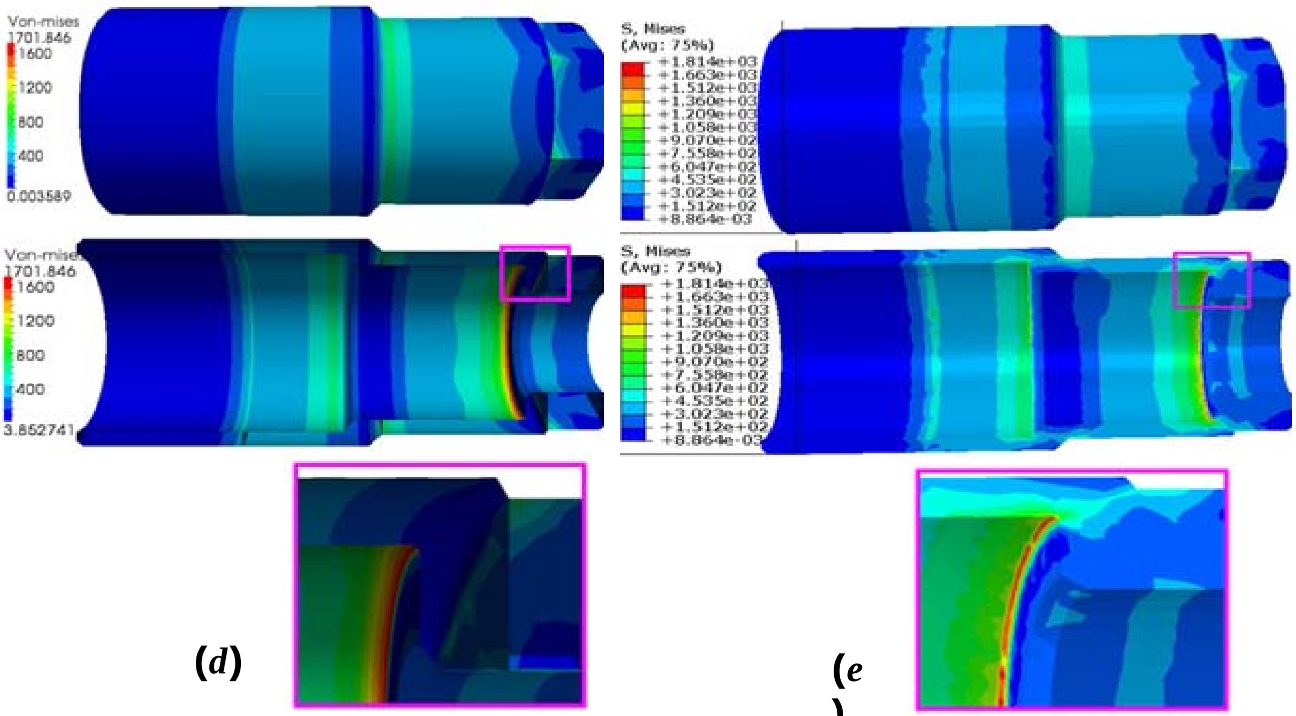
Stress concentration

■ Nozzle cap nut



(*a*) (*b*) (*c*)

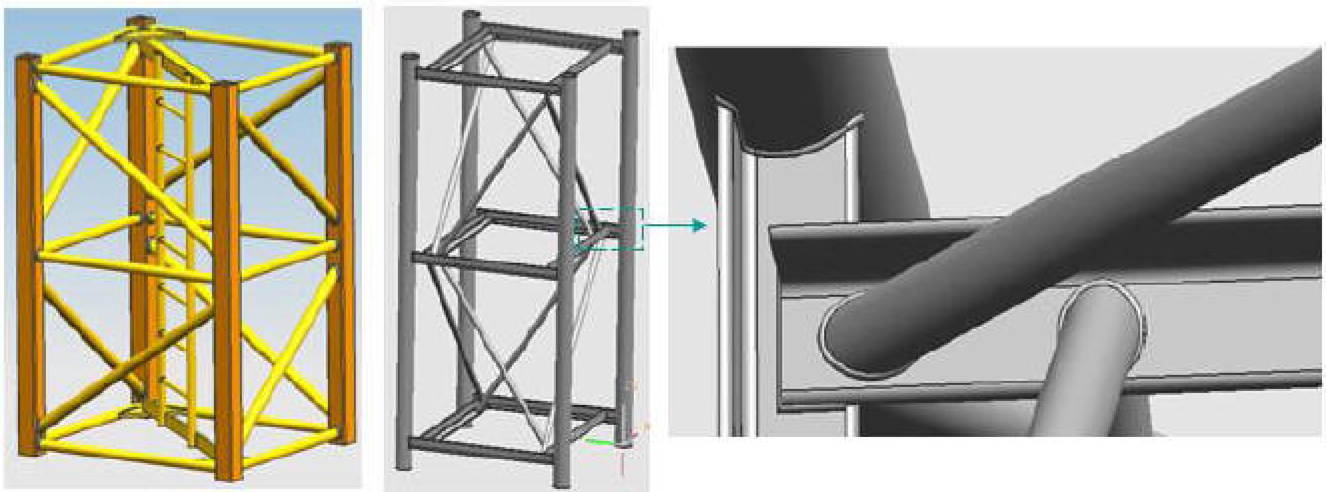
算法	节点数目	最大应力 (MPa)
BFM	5422	1710.15
	8014	1701.85
	14455	1701.85
FEM	35098	1698
	89548	1762
	136733	1814

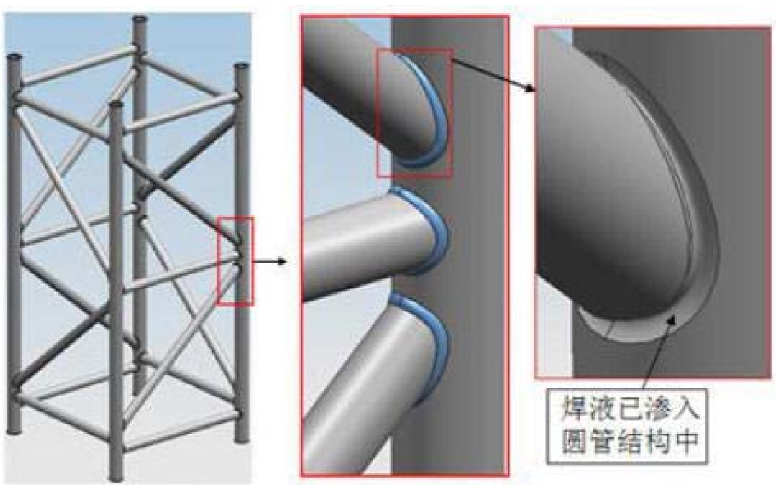


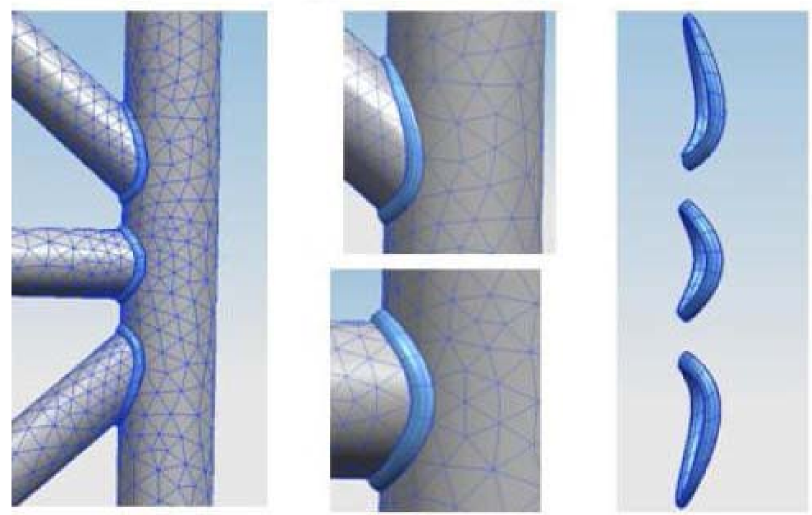
(*d*) (*e*)

Stress concentration

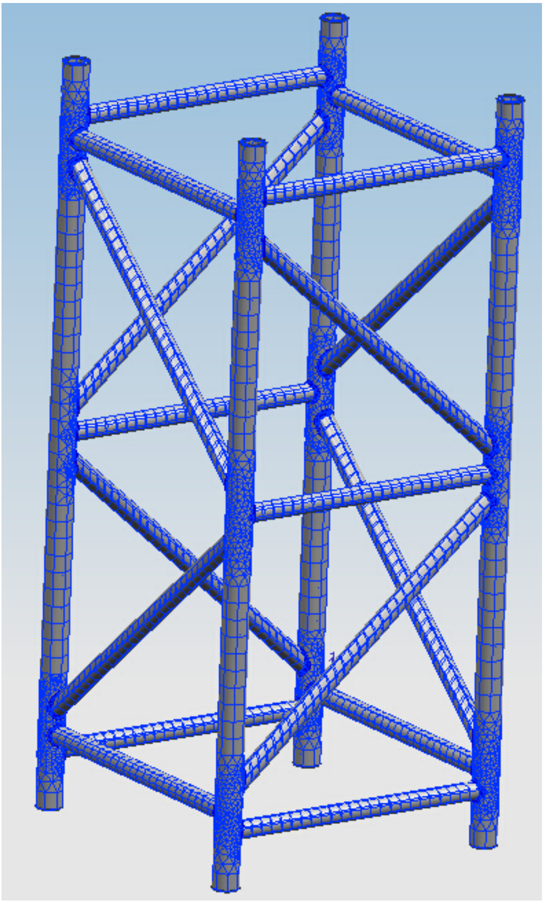
- **Welded Steel Frame**

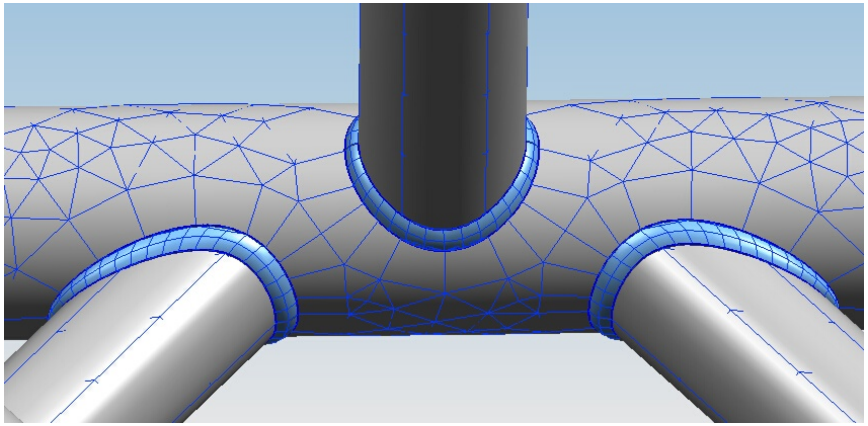


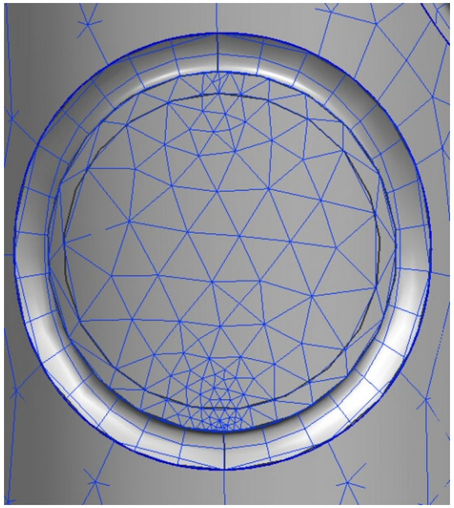


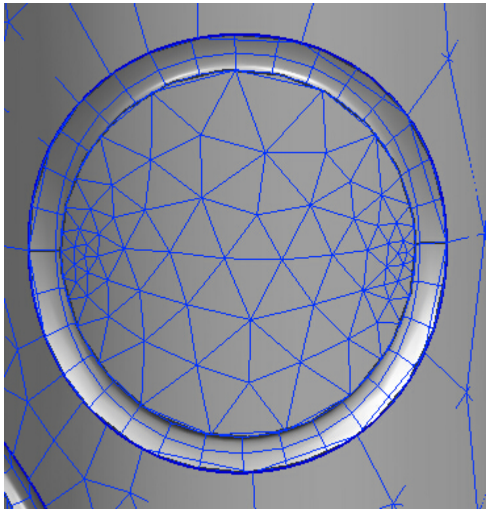


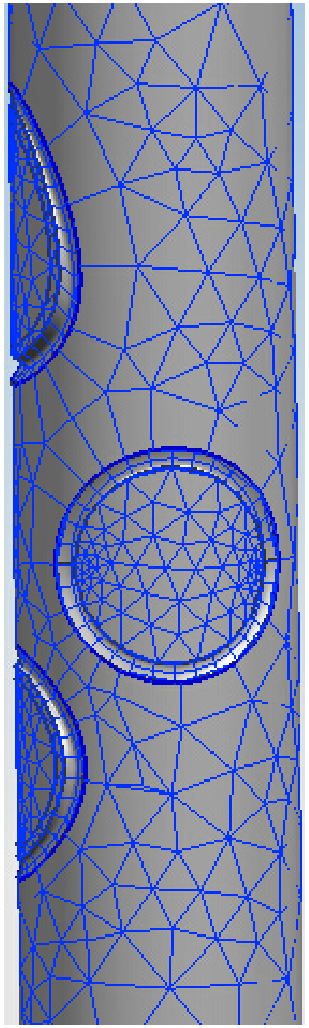
Stress concentration











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